# Sampling Distributions (Page 331-343, Chapter 13)

**TODAY YOU WILL BE ABLE TO…**

* Describe a binomial experiment (the binomial setting)
* Describe binomial distributions and its parameters
* Calculate binomial probabilities
* Calculate and interpret the binomial mean and standard deviation
* Calculate the Normal approximation to binomial distributions

Like the Normal Distribution, the binomial distribution has a pattern that allows us to make inferences about populations that we know follow this distribution.

**RECALL**

* A **random variable** is a rule that assigns one (and only one) numerical value to each simple event of an experiment.

Consider tossing a fair coin 3 times. What is the sample space (possible outcomes)?

S={TTT, HTT, THT, TTH, THH, HTH, HHT, HHH}

Define the random variable X to be the number of heads obtained.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X=no. of heads | 0 | 1 | 2 | 3 |
| Outcomes | TTT | HTT, THT, TTH | THH, HTH, HHT | HHH |

* The **probability distribution** of a random variable gives the values of the random variable and their probabilities. Note that P(heads) = 0.5 and P(tails) = 0.5 so each outcome is equally likely…

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X=no. of heads | 0 | 1 | 2 | 3 |
| Probability | 1/8 | 3/8 | 3/8 | 1/8 |

* P(A or B) = P(A) + P(B) when events A and B are disjoint.
* P(A and B) = P(A) × P(B) when events A and B are independent.

**THE BINOMIAL EXPERIMENT**

A **binomial experiment** occurs when all of the following criteria are met. The experiment of tossing a coin three times meets the binomial criteria.

1. There are a **fixed number of observations** denoted by n.
   * How many observations were in the previous coin tossing experiment?
2. The n observations are all **independent**, i.e., knowing the result of one observation does not change the probabilities we assign to other observations.
   * Is tossing a coin independent of other coin tosses?
3. The various combinations of possible outcomes are disjoint.
   * Is one combination disjoint of another? Ask: Can you obtain HHH and HTH for the same set of three coin tosses?
4. Each observation has ONLY **two possible outcomes**.
   * What are the possible outcomes of tossing a coin? How many?

When counting a particular outcome, such as the number of heads, the outcome we are counting is called a “**success**” while the other outcome is called a “**failure**.”

1. The probability of **success**, call it ***p***, is the same for each observation. The probability of **failure**, for each observation, is ***1-p***.
   * Is the probability of tossing heads the same each time you toss the coin?

**THE BINOMIAL DISTRIBUTION**

The parameter **p** is called the **binomial (or population) proportion**. We say that the random variable X has a binomial distribution with parameters n and p.

In very many repetitions of binomial experiments…

* The average count of successes, i.e., the mean of a binomial random variable, is   
  μ = np
* The variance is σ2 = np(1 – p)
* The standard deviation is σ = 

**BINOMIAL PROBABILITIES**

Define the random variable X to be the number of heads obtained when tossing a coin three times. What is the probability of getting **two** heads?

With heads being the outcome of interest, the “success”, define P(heads) to be 0.51.

p = 0.51

1-p = 0.49

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X = no. of heads | 0 | 1 | 2 | 3 |
| Probability |  |  |  |  |

Using the probability formula for independent events…

P(X = 2) is

P(**heads**) × P(**heads**) × P(tails) = 0.51 × 0.51 × 0.49 = 0.1274

OR

P(tails) × P(**heads**) × P(**heads**) = 0.49 × 0.51 × 0.51 = 0.1274

OR

P(**heads**) × P(tails) × P(**heads**) = 0.51 × 0.49 × 0.51 = 0.1274

**NOTICE…**

P(X = 2) =

[**(0.51)2 × (0.49)1**] + [**(0.51)2 × (0.49)1**] + [**(0.51)2 × (0.49)1**] =

0.1274 + 0.1274 + 0.1274

**NOTICE ALSO…**

**3** × [**(0.51)2 × (0.49)1**] =

\*\*\*\*\*\*\*\*\*\* “3” is the number of ways of arranging two heads! \*\*\*\*\*\*\*\*\*\*

**What happens when there are too many combinations to list?**

The number of ways of arranging k successes among n observations is given by the **binomial coefficient**, denoted and pronounced “n choose k,”

for k = 0, 1, 2, 3, …, n.

is NOT a fraction!

can also be written nCk, and this is how the binomial coefficient is represented on some calculators!

***Example 1***

On average, a basketball player makes 80% of her free throw shots. Consider the player taking 5 free throw shots and assume the outcome of each shot is independent of the outcome of any of the other shots.

1. Examine the four conditions for a binomial experiment.

* The experiment consists of n = \_\_\_\_\_\_ trials (free throw attempts).
* The player will either make (success) or miss (failure) each shot.
* The probability of making a shot on each attempt is approximately p = \_\_\_\_\_\_\_.
* The outcome of each shot is independent of the outcome of any of the other shots.

1. Define the binomial random variable.

1. Specify (in words) the distribution of X.

1. Write the formula for the probability distribution function of X.

1. Write the probability distribution in tabular form.

|  |  |  |
| --- | --- | --- |
| **PDF of X** | |  |
| **x** | **p(x)** |  |
| 0 |  | ← p(0) = P( X = 0 ) = 1⋅ ( 0.80 )0 ⋅ ( 0.20 )5 |
| 1 |  | ← p(1) = P( X = 1 ) = 5⋅ ( 0.80)1 ⋅ ( 0.20 )4 |
| 2 |  | ← p(2) = P( X = 2 ) = 10⋅ ( 0.80)2 ⋅ ( 0.20 )3 |
| 3 |  | ← p(3) = P( X = 3 ) = 10⋅ ( 0.80)3 ⋅ ( 0.20 )2 |
| 4 |  | ← p(4) = P( X = 4 ) = 5⋅ ( 0.80)4 ⋅ ( 0.20 )1 |
| 5 |  | ← p(5) = P( X = 5 ) = 1⋅ ( 0.80)5 ⋅ ( 0.20 )0 |
| Total | **1.0000** |  |

1. Find the mean and the standard deviation of X. Interpret these numbers.

***Example 2***

A player wins a certain game of chance about 32% of the time. Suppose the player plays the game 6 times. Find the probability that the player wins the game exactly twice. Begin by defining the random variable X and specifying the distribution of X.

***Example 3***

A fair four-sided die is tossed 8 times. Find the probability of observing no more than 1 four. Begin by defining the random variable X and specifying the distribution of X.

***Example 4***

A poorly prepared student is guessing at a 10-question multiple choice exam. Each question has 5 selections. Find the probability of observing at least 8 correct answers. Begin by defining the random variable X and specifying the distribution of X.

***Example 5***

Approximately 10% of the world’s population is left-handed. Consider 25 randomly selected people. Define the random variable X to be the number of lefties in the sample.

1. Specify (in words) the distribution of X.
2. Find the probability of observing no more than 4 lefties in the sample.
3. Find the probability of observing at least 5 lefties in the sample.
4. Find the probability of observing between 2 and 5 lefties in the sample.
5. Find the mean and standard deviation of X.

**NORMAL APPROXIMATION TO BINOMIAL DISTRIBUTIONS**

Suppose that a count X has the binomial distribution with n observations and success probability p.

When n is large, the distribution of X is approximately Normal with mean μ = np and standard deviation σ = . That is X has the distribution N(np, ).

This is nice, since we really do not want to explicitly calculate binomial probabilities when n > 100!

How large should n be? For binomial distributions, use the Normal approximation when n is so large that …

np ≥ 10

and

n(1-p) ≥ 10

***Example 6***

If 2% of people have red hair, what is the probability that more than 15 in a random sample of 500 people have red hair?

***Example 7***

If 34% of professors at a university are women, what is the probability that fewer than 60 in a random sample of 250 professors are women?